Problem 13

Three digital clocks A, B, and C run at different rates and do not have simultaneous readings of zero. Figure 1-6 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, B reads 25.0 s and C reads 92.0 s.) If two events are 600 s apart on clock A, how far apart are they on (a) clock B and (b) clock C? (c) When clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

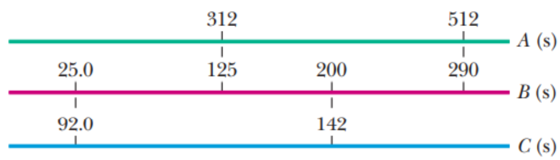

Figure 1-6 Problem 13.

Solution

Part (a)

Convert from Clock A to Clock B.

$$600 \text{ s on Clock A} = 600 \cancel{\text{ s on Clock A}} \times \frac{(290-125) \text{ s on Clock B}}{(512-312) \cancel{\text{ s on Clock A}}} = 495 \text{ s on Clock B}$$

Part (b)

Convert from Clock A to Clock B to Clock C.

$$600 \text{ s on Clock A} = 600 \cancel{\text{ s on Clock A}} \times \frac{(290-125) \cancel{\text{ s on Clock B}}}{(512-312) \cancel{\text{ s on Clock A}}} \times \frac{(142-92.0) \text{ s on Clock C}}{(200-25.0) \cancel{\text{ s on Clock B}}}$$

$$\approx 141 \text{ s on Clock C}$$

Part (c)

The aim here is to write a formula for the time on Clock B, given the time on Clock A. Let y be the time on Clock B, and let x be the time on Clock A.

$$y = mx + b$$

Two points on this line are $(312, 125)$ and $(512, 290)$.

$$125 = 312m + b \quad (1)$$

$$290 = 512m + b \quad (2)$$

Subtract the respective sides to eliminate b.

$$-165 = -200m$$

Solve for m.

$$m = \frac{33}{40}$$

Substitute this value into either equation (1) or equation (2) to determine b.

$$125 = 312\left(\frac{33}{40}\right) + b \quad \rightarrow \quad b = -\frac{662}{5}$$

The equation of the line is then

$$y = \frac{33}{40}x - \frac{662}{5}.$$

Therefore, when Clock A is at 400 s the time on Clock B is

$$\begin{aligned} y &= \frac{33}{40}(400) - \frac{662}{5} \\ &= \frac{988}{5} \text{ s} \\ &\approx 198 \text{ s}. \end{aligned}$$

Part (d)

The aim here is to write a formula for the time on Clock B, given the time on Clock C. Let y be the time on Clock B, and let x be the time on Clock C.

$$y = mx + b$$

Two points on this line are $(92.0, 25.0)$ and $(142, 200)$.

$$25.0 = 92.0m + b \tag{3}$$

$$200 = 142m + b \tag{4}$$

Subtract the respective sides to eliminate b.

$$-175 = -50m$$

Solve for m.

$$m = \frac{7}{2}$$

Substitute this value into either equation (3) or equation (4) to determine b.

$$25.0 = 92.0\left(\frac{7}{2}\right) + b \quad \rightarrow \quad b = -297$$

The equation of the line is then

$$y = \frac{7}{2}x - 297.$$

Therefore, when Clock C is at 15 s the time on Clock B is

$$\begin{aligned} y &= \frac{7}{2}(15) - 297 \\ &= -\frac{489}{2} \text{ s} \\ &\approx -245 \text{ s}. \end{aligned}$$